THE COEVOLUTION OF GROUP SIZE AND LEADERSHIP:
AN AGENT-BASED PUBLIC GOODS MODEL
FOR PREHISPANIC PUEBLO SOCIETIES

TIMOTHY A. KOHLER
Department of Anthropology,
Washington State University,
Pullman, WA 99164-4910
Santa Fe Institute/Crow Canyon Archaeological Center

DENTON COCKBURN
School of Computer Science,
University of Windsor, 401 Sunset Avenue,
Windsor, ONT N9B-3P4

PAUL L. HOOPER
Department of Anthropology,
University of New Mexico,
Albuquerque, NM 87131

R. KYLE BOCINSKY
Department of Anthropology,
Washington State University,
Pullman, WA 99164-4910

ZIAD KOBTI
School of Computer Science,
University of Windsor, 401 Sunset Avenue,
Windsor, ONT N9B-3P4

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We present an agent-based model for voluntaristic processes allowing the emergence of leadership in small-scale societies, parameterized to apply to Pueblo societies of the northern US Southwest between AD 600 and 1300. We embed an evolutionary public-goods game in a spatial simulation of household activities in which agents, representing households, decide where to farm, hunt, and locate their residences. Leaders, through their work in monitoring group members and punishing defectors, can increase the likelihood that group members will cooperate to achieve a favorable outcome in the public-goods game. We show that under certain conditions households prefer to work in a group with a leader who receives a share of the group’s productivity, rather than to work in a group with no leader. Simulation produces outcomes that match reasonably well those
known for a portion of Southwest Colorado between AD 600 and 900. We suggest that for later periods a model incorporating coercion, or inter-group competition, or both, and one in which tiered hierarchies of leadership can emerge, would increase the goodness-of-fit.

**Keywords**: Public-goods games; agent-based simulation; spatial simulation; emergence of leadership; Neolithic societies; Pueblo society; Southwestern archaeology; southwestern Colorado.

1. Introduction

In this paper, we develop and explore a computational model for one pathway by which leadership might emerge in small-scale societies. Our model is based on an evolutionary game theoretic model put forward by Hooper *et al.* [12] of a voluntaristic process in which members of a society would prefer to live in a more hierarchically structured group, than in a more egalitarian one, if leaders can reduce the likelihood of failures in cooperation due to free-riding or lack of coordination. Their model suggested that choosing to work under the supervision of a leader becomes an optimal decision for members of a society when cooperation as a group is potentially profitable, but their group size exceeds that in which leaderless cooperation is viable.

Using an agent-based approach, we examine the power and limitations of this general analytic model by mapping it into a specific place and time in which the archaeological record suggests the emergence of hierarchically organized groups, and consequently, of leaders. This will allow us to assess the model’s empirical plausibility for our area: can this model work, with reasonable parameter values, within the time and in a space in which real hierarchies seem to have been able to evolve? Richerson *et al.* [31] argue that change in social institutions is difficult, and may come to be a limiting factor in population growth. How much of the “work” in creating hierarchical societies in this time and place can this model carry? Eventually, we intend to confront our empirical record from southwestern Colorado with several different models for the formation of hierarchical societies, and then adjudicate among these using more formal goodness-of-fit approaches as illustrated by [15, 21, 4]. Treated in that way, the model can provide us with useful information about the local archaeological record itself.

1.1. Public goods and common goods

The analytical model of Hooper *et al.* [12] focuses on how individuals should behave in a public-goods game. We will substitute the behavior of households for that of individuals. Consider for example Ellen who greatly enjoys listening to her local National Public Radio affiliate. Should she contribute to its annual fund drive? If she does (cooperate), she helps ensure that those programs will be there in the future, and she perhaps adds very slightly to their potential quality. But if she defects — if she free-rides on the contributions of others — she knows there’s a good chance that the station can keep going anyway. Here the station’s broadcast
represents a costly benefit from which she cannot be excluded — a “public good”. Her annual dilemma is a repeated public goods game. The Hooper et al. model [12] adds the possibility of monitoring members of a social group and punishing free-riders, an option clearly not available in this example, possibly to the regret of NPR station managers.

A public good therefore is something that is non-rivalrous (Ellen’s listening does not diminish another’s ability to do so) and non-excludable. (It is impractical to prevent anyone within range of the signal from hearing the broadcast.) Defense is the classic public good. Groups who construct reservoirs or irrigation networks are solving the closely related common-goods problem, assuming that no one is excluded from use of the water they provide, because those resources have a rivalrous aspect; water used by one household is unavailable to another. Here we will consider both common and public goods simply as public goods. Most of the relevant situations in the reference system seem to involve public goods, and even goods that are technically common rather than public (such as water in a reservoir) are large relative to individual usage. More importantly, solutions to both sorts of problems require similar coordination and cooperation in groups ordinarily much larger than one’s immediate kin group. In the archaeological record referenced in this model, we interpret defensive works like site walls, community ritual structures like great kivas and plazas, water-storage devices, and other works apparently built and used by more than one household as successful solutions to these classes of problems. Some other public goods, for example, the rabbits caught when a community puts on a drive, will be harder to identify archaeologically (but see [28] for an attempt to identify communal hunting elsewhere in the Southwest), so we likely underestimate the importance of public goods in the archaeological record.

A critical characteristic of a public-goods game is that, if everyone participates, each can get out more than she puts in: cooperation allows positive returns to scale ([34, p. 14]). However, members of a group also have a strong temptation to free ride on the contributions of others, since, if they can get away with it, their returns will be even higher.

1.2. Key characteristics of the later prehistory in the central Mesa Verde Region

Let us now turn to a brief discussion of the setting in which we will place this model. The archaeological record of the US Southwest between about AD 600 and AD 1200 strongly suggests four Big Facts that make it attractive for this purpose. First, this period marks the early, high-population-growth phase of a Neolithic Demographic Transition as defined by [5, 19]. This population growth is super-regional [17, 20], and the way various subregions participate in it is somewhat variable. For the last decade, the Village Ecodynamics Project (VEP) has been intensively examining the famous and densely occupied portion of the Pueblo Southwest, in Southwest Colorado, known as the central Mesa Verde Region (VEP I on Fig. 1); [37]. In this area,
the long-term regional process of population growth is damped by low-frequency cool temperatures and short summers that were unfavorable for maize agriculture, the staple for these farmers. These unfavorable conditions were centered on the mid-AD 900s [39], and contributed to the interrupted growth trajectory shown in Fig. 2. Occupation of this area by farmers terminated in the late-AD 1200s in a cascade of unfavorable circumstances that included at least high- and low-frequency deterioration of climates for maize farming in the context of previous depression of large game and very high dependence on turkey (often fed maize) [30] for protein, and concurrent immigration of farmers retreating from more northern portions of the Southwest. These factors resulted in widespread violence in the central Mesa Verde area [20, 22]. The northern Rio Grande region of New Mexico then became the principal (though not sole) locus of Pueblo occupation [11].

The second Big Fact is that overall, these increasing populations were distributed among increasingly large groups. Habitation sites in the northern Southwest tend to be obvious on the ground and well-bounded in space, and these sites generally tend to get bigger through the period we study. It is common in the Pueblo world to find a rather strong positive relationship between the total regional population and the proportion of people living in large sites, so Facts 1 and 2 are
Fig. 2. Three estimates of the momentary population through time, in households, for the VEP I study area (from [37]). We prefer the middle estimate.

Connected. Increasing site size is not the only sense in which social groups increased in size in this area, though. In most periods there are also people living in smaller habitations around the larger centers who need to be counted as part of communities centered on the larger sites. The sites participating in such communities are identified by some degree of spatial clustering, with fall-offs in site density representing boundaries between communities. Mahoney et al. [25] suggest that “residential community” size in several well-studied subsets of the Mesa Verde Region changed from between 35 and about 190 people in the early AD 900s to between 240 and about 390 people in the late 1200s. (Recent research suggests that their figures may be a little too low; Kohler and Varien [19] produced rank-size plots for selected periods in the VEP study area showing that a few individual sites have ca. 100 households, and therefore perhaps 500 people, during both the first and second peaks of occupation, even before the contributions of their supporting communities are accounted for.) Mahoney and her colleagues suggest that communities, as a level of social organization, served to protect claims to resources, especially productive farmlands.

Here we are interested in knowing how the size of the largest social group within which one regularly collaborates changed through time. A slightly more precise way of saying this is, what is the size of the largest social group within which
public-goods and common-goods problems were regularly faced, and often solved? The long-term trend throughout this period is not only for the communities to get bigger, as we noted, but also for these communities to coordinate their activities with increasingly more communities. This tendency is especially noticeable during the 250 years of Chacoan growth, which peaked during the first half of the AD 1100s. During that period, most of the eastern Pueblo Southwest was either under Chaco’s political control, or was at least coordinating its ritual activities with those at Chaco [23, pp. 230–242], a scale involving many dozens of communities.

The third Big Fact that is particularly relevant here is that during most of the AD 600–1300 period, there was a tendency for these increasingly large groups to become more hierarchical. In general hierarchy is rather subtly expressed in the Pueblo archaeological record. During portions of the Pueblo I period (in the AD 700s and 800s) villages in the VEP area had some households, probably including the heads of successful lineages, who lived in substantially larger pithouses than others and likely coordinated ritual and hunting activities and perhaps social usage of stored foods [13, 18, 33]. This rather modest social differentiation apparently laid the foundation for a much-less-subtle expression of hierarchy in the following Pueblo II, Chaco-dominated period. Archaeologist Steve Lekson [23] considers the great houses that typify the central Chacoan sites to be “palaces” of “kings”. Although this view is too extreme for many southwestern archaeologists, the degree of social differentiation reconstructed from a comparison of burials at the largest of the Chaco great houses, Pueblo Bonito, and the small houses in Chaco Canyon is extreme in comparison with the rest of the Pueblo world [27].

Also relevant to our discussion here is a final, fourth Big Fact: the increasing importance of public goods through the 700-year sequence we study. These Chaco leaders in the “downtown” big houses, as well as leaders in lower-ranked communities throughout the eastern Pueblo area, were evidently able to mobilize a great deal of labor for public uses of various types, including building roads, reservoirs, defensive walls, massive civic-ceremonial structures such as great kivas, and presumably for engaging in conflict and long-distance hunting. Some of these activities and features, such as great kivas, appear in the first (AD 600–900) population cycle in our area [1], but others, such as reservoirs, roads, defensive walls, and great houses, are either completely or mostly restricted to the second population cycle ([24, pp. 272–289, 38]). In general we consider all of Glowacki and Ortman’s [10] “group-assembly features” and “controlled-access features” as providing public goods.

So, here are the four Big Facts: through time in our area we see regional population growth, the appearance of larger sociopolitical groups, the emergence of more strongly marked political/religious hierarchies, and the increasing prominence of public goods. Are these changes independent and coincidental? Given just this one sequence, that might be possible. However, these trends appear in correlated fashion in many well-studied Neolithic sequences, as studies from around the world testify [3], making the odds of coincidence remote. We now briefly describe an analytical model that connects these facts in a compelling causal framework. We then
agentize and spatialize that model in the VEP area, and discuss its strengths and weaknesses with respect to the local archaeological record. We close by considering what other factors would need to be considered to develop a complete model for the emergence and decline of hierarchy in the eastern Pueblo Southwest.

2. The Hooper et al. Model for the Evolution of Leadership

Hooper, Kaplan, and Boone [12] argue that leadership can emerge in cooperative groups, under circumstances that include expansion of group size, if leaders can “reduce the likelihood that cooperation fails due to free-riding or coordination errors and that under some circumstances, individuals would prefer to cooperate in a group under the supervision of a leader who receives a share of the group’s productivity than to work in an unsupervised group” [12, p. 633]. A repeated public-goods game (Table 1) provides the framework for their model. In this game, groups of size \( n \) are randomly drawn from a very large population. In each round, every group member can either contribute to the public good at a personal cost \( c \), or refuse to contribute (defect). Each group member, whether or not she contributed, receives a benefit \( b/n \) from every contributor in the group. A parameter \( w \) represents the likelihood that the game will be played again. When play ceases, agents reproduce in proportion to

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Base values used by [12]</th>
<th>Values used here (expressed in units of days, kg, a one-day’s consumption of maize for an average household)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>Benefit available from the public good</td>
<td>2</td>
<td>emergent, maximally 73</td>
</tr>
<tr>
<td>( c )</td>
<td>Cost of contributing to the public good</td>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>( n )</td>
<td>Number of individuals in the group</td>
<td>experimentally imposed</td>
<td>emergent</td>
</tr>
<tr>
<td>( s )</td>
<td>Cost imposed on defectors (sanction or punishment)</td>
<td>1.5</td>
<td>56</td>
</tr>
<tr>
<td>( \hat{s} )</td>
<td>Cost imposed on non-taxpayers</td>
<td>1.5</td>
<td>56</td>
</tr>
<tr>
<td>( c_m )</td>
<td>Cost of monitoring one group member</td>
<td>0.05</td>
<td>2, 4, 6</td>
</tr>
<tr>
<td>( c_s )</td>
<td>Cost of punishing one defector</td>
<td>0.15</td>
<td>11</td>
</tr>
<tr>
<td>( c_{\hat{s}} )</td>
<td>Cost of punishing one non-taxpayer</td>
<td>0.15</td>
<td>11</td>
</tr>
<tr>
<td>( t )</td>
<td>Fraction of benefits from cooperation appropriated by leader</td>
<td>( c_m/b &gt; t &lt; 1 - c/b ) (0.025)</td>
<td>( c_m/b &gt; t &lt; 1 - c/b ) (0.05)</td>
</tr>
<tr>
<td>( \hat{t} )</td>
<td>Fraction of benefits from cooperation appropriated by non-leader</td>
<td>( c_m/b &gt; t &lt; 1 - c/b ) (0.5)</td>
<td>( c_m/b &gt; t &lt; 1 - c/b ) (0.5)</td>
</tr>
</tbody>
</table>

(assigning \( b = 73 \))

The Coevolution of Group Size and Leadership
their accumulated pay-offs in all rounds and die. Hooper and his colleagues propose three scenarios, involving differing populations of strategies. Here we give a very brief outline and summary of the outcomes of the study; for details see [12].

2.1. Free unilateral contribution

In the simplest scenario there are only two strategies: Always cooperate (that is, contribute to the public good: ALLC), and always defect (ALLD). The pay-off to cooperating if there are \( x \) other cooperators in the group is

\[
V(\text{ALLC} \mid x) = \frac{(x+1)b}{n} - c - w
\]

(1)

whereas the defector receives

\[
V(\text{ALLD} \mid x) = \frac{x b}{n} - w
\]

(2)

The pay-off to cooperate is greater than the pay-off to defect when

\[
b/n > c
\]

(3)

Free unilateral contribution then can survive as a strategy only when the benefits to cooperation are very high relative to its costs, and group size is very small (on the order of just a couple of people in their base parameter settings). Clearly some other set of strategies must be at work to allow collective action at the larger group sizes we see in the archaeological record.

2.2. Mutual monitoring and punishment

A great deal of theoretical work in the last decade [7, 6] has proposed that under certain circumstances cooperation can be sustained by punishment. Hooper and colleagues introduce a mutual monitoring (MM) strategy that always cooperates and pays a cost \( c_m \) to monitor the contributions of each other group member in each round. Any individual who fails to contribute to the public good will be punished by MM at a cost to MM of \( c_s \), reducing the defector’s pay-off by \( s \) immediately following the round. They also replace the ALLD strategy of the previous game with Reluctant Cooperators (RC), who defect until they are punished and then cooperate in subsequent rounds, so long as the punishment exceeds the net cost of contributing to the public good. In other respects the game is the same.

Without detailing the pay-offs to each type we can summarize the main long-term results for specific combinations of parameter values via Fig. 3, which reproduces Fig. 4 from [12, p. 637]. When group sizes are extremely small, reproducing the conditions in expression (3), ALLC dominates. At slightly larger group sizes Mutual Monitors invade and quickly become almost half of the population. However, as group sizes continue to increase, MMs begin to lose out to RCs, and this
Fig. 3. Equilibrium frequencies of pure cooperators (darker gray), mutual monitors (black), and reluctant cooperators (lighter gray) as functions of group size in a non-hierarchical population. The three frequencies are stacked and sum to one. Top: Baseline parameter values \( b = 2; c = 1; w = 0.9; c_m = 0.05; c_s = 0.15; s = 1.5; \) and \( V_0 = 40. \) Bottom: Doubled costs of monitoring and sanctioning: \( c_m = 0.1 \) and \( c_s = 0.3. \) From [12, p. 637].

happens more rapidly as costs for monitoring and sanctioning increase. Of course, when MMs disappear, the RC strategy is to defect. The Mutual Monitoring strategy thus appears to fail to sustain cooperation as group sizes increase beyond a certain point (a point reached in groups of only 10–15 people, using the balances of costs and benefits given by the baseline parameters in [12]).
2.3. Cooperation reinforced by the effort of a leader

In a third scenario, Hooper and his colleagues add the possibility that individuals may be either willing (H), or unwilling (NH), to join a hierarchical group, defined as a group that “elects” a leader (L) who receives a share of the group productivity as a tax in return for monitoring other group members to make sure that they contribute to the public good, and that they pay a tax to support the leader. The leader incurs a cost for monitoring and if necessary punishing free riders (H.RC) and reluctant taxpayers (H.RT). Individuals unwilling to work in such a group continue to play the three strategies (NH.ALLC, NH.RC, and NH.MM) defined just above. (Should agents with a hierarchical preference be leaderless, they will also play these strategies.) Hierarchical groups with more than one potential leader will elect whichever offers the lowest tax rate $t$. Non-leaders may be either willing taxpayers (H.T), or reluctant tax payers (H.RT); like reluctant cooperators, reluctant taxpayers do not pay the tax until punished, and then always pay in subsequent rounds. Hierarchic types willing to lead (L) but not chosen, because another H.L is offering a lower tax rate in that group, receive the regular hierarchical group member’s pay-off.

Combining the independent traits of willingness to live in a hierarchic group (NH versus H), willingness to cooperate in the public-goods game (NH.ALLC versus NH.RC and H.ALLC versus H.RC), willingness to pay taxes if in a hierarchic group (H.T versus H.RT) and willingness to be a leader if in a hierarchic group (H.L versus H.UL) yields the three NH types and 8 H types whose pay-offs are given in Table 2.

It emerges from the dynamics of this third model, in comparison with those of the first and second models, that

the appointment of a supervising leader who receives a share of group productivity can be a robust and mutually preferred solution to the problem of free-riding in cooperative groups. . . . this leadership regime becomes preferable to the alternatives of non-cooperation and leaderless cooperation when there are significant returns to scale, yet the size of the group demanded by the activity is too large for individuals to be motivated to either contribute to the public good or sanction free-riders on their own. The model also predicts that leadership will be preferable to leaderless cooperation when the cost of monitoring and sanctioning fellow group members is high enough that individuals would be unwilling to enforce cooperation on their own, but not so high that the gains from cooperation cannot cover the leader’s enforcement costs [12, p. 642].

3. Agentizing and Spatializing the Hooper et al. Model

We add a version of this public goods game to the operation of the current version of the “Village” agent-based model (ABM), built on the RepastJ (Repast 3.0)
Table 2. Agent types and approximate pay-offs related to their participation in the public-goods game. Pay-offs are approximate since it cannot be known in general whether reluctant cooperators (in both group types) or reluctant taxpayers (in the hierarchically inclined groups) will need to be punished in any given year. Pay-offs to leaders refer to agents actually acting as leaders; potential (latent) leaders receive pay-offs appropriate to their actions as regular members of their hierarchical group.

<table>
<thead>
<tr>
<th>Type</th>
<th>Symbol (Figs. 8 and 10)</th>
<th>Approximate pay-offs (revised from [12, pp. 636-641])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-hierarchically inclined agents (o)</td>
<td>grey o</td>
<td>[ V(ALLC</td>
</tr>
<tr>
<td>NH.ALLC (not hierarchic, always cooperate)</td>
<td>grey o</td>
<td>[ V(ALLC</td>
</tr>
<tr>
<td>NH.MM (not hierarchic, mutual monitor)</td>
<td>pink o</td>
<td>[ V(MM</td>
</tr>
<tr>
<td>NH.RC (not hierarchic, reluctant cooperator)</td>
<td>black o</td>
<td>[ V(RC</td>
</tr>
<tr>
<td>Hierarchically inclined agents (leaders square, non-leaders +)</td>
<td>u = fraction of pure cooperators; v = fraction of willing taxpayers; ( y = ) fraction of individuals willing to lead</td>
<td></td>
</tr>
<tr>
<td>H.ALLC.T.L (hierarchic, always cooperate, taxpayer, leader)</td>
<td>turquoise square</td>
<td>[ V(L</td>
</tr>
<tr>
<td>H.ALLC.T.UL (hierarchic, always cooperate, taxpayer, nonleader)</td>
<td>rose +</td>
<td>[ V(H.ALLC.T</td>
</tr>
<tr>
<td>H.ALLC.RT.L (hierarchic, always cooperate, reluctant taxpayer, leader)</td>
<td>green square</td>
<td>as for H.ALLC.T.L</td>
</tr>
<tr>
<td>H.ALLC.RT.UL (hierarchic, always cooperate, reluctant taxpayer, nonleader)</td>
<td>green +</td>
<td>[ V(H.ALLC.RT</td>
</tr>
<tr>
<td>H.RC.T.L (hierarchic, reluctant cooperator, taxpayer, leader)</td>
<td>orange square</td>
<td>as for H.ALLC.T.L</td>
</tr>
<tr>
<td>H.RC.T.UL (hierarchic, reluctant cooperator, taxpayer, nonleader)</td>
<td>aqua circle</td>
<td>[ V(H.RC.T</td>
</tr>
<tr>
<td>H.RC.RT.L (hierarchic, reluctant cooperator, reluctant taxpayer, leader)</td>
<td>red +</td>
<td>as for H.ALLC.T.L</td>
</tr>
<tr>
<td>H.RC.RT.UL (hierarchic, reluctant cooperator, reluctant taxpayer, nonleader)</td>
<td>blue +</td>
<td>[ V(H.RC.RT</td>
</tr>
</tbody>
</table>
libraries. A slightly older version of this simulation (built on the Swarm libraries) is documented in detail in [21] and here we give only a very brief overview of its operation.

3.1. **Summary of the Village ABM**

The Village Ecodynamics Project is a long-running research stream currently supported by NSF’s Dynamics of Coupled Natural and Human Systems program. It has both an empirical emphasis, including the organization and transformation of large databases of archaeological sites collected over many decades [26] along with some new archaeological survey [10], and a modeling emphasis that currently revolves around the ABM we call “Village.” The purpose of Village is to simulate household-level choices about where to live, farm, hunt, and collect firewood and water in a landscape that is changing yearly due to human impact on these resources and in response to high-frequency climatic variability as interpreted from tree-rings. We model the calories and time spent in a variety of activities surrounding these basic subsistence chores, and also model generalized reciprocal exchange, and balanced reciprocal exchange (as characterized by [32]), in the currencies of calories (maize) and protein (meat), without cross-currency exchanges.

Pueblo households are agents, and utilize the portion of southwestern Colorado shown in Fig. 1, which is about 1800 km$^2$ in size (modeled as 200 × 227 4-ha cells). Households implement an approximately optimal strategy in determining where to locate their activities. Running the model allows us to make quantitative estimates of the degree to which these populations depressed various resources, and the carrying capacities and time budgets for humans on this landscape under various parameter combinations. Comparison of its output with the known locations of households on this landscape during the 14 periods we identify between AD 600 and 1300 allows us to determine the extent to which spatial behaviors of Pueblo farmers were optimal, the record of change in degree of optimality through time, and the varying importance of various parameters through time in influencing settlement behavior.

With the weak exceptions of the economic networks created by balanced reciprocal exchange among households, and the kinship networks used by generalized reciprocal exchange, our agents — up to this point — have not formed or participated in groups. We wanted to tackle social complexity after learning how to build agents that could successfully negotiate the ecological complexity they face. A high priority for us now is to study processes of decision-making above the level of the household, including group formation, inter-group competition, and the evolution of leadership. This paper represents our first foray into this cluster of research topics. It is important to note that our approach allows us to study these topics in the context of a semi-realistic, quantitatively expressed and dynamic environment onto which spatial patterns are deductively generated from the model, rather than as a conceptual and usually inductive process, which has been the more traditional archaeological approach to these topics.

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3.2. The structure of the public-goods ABM

The ABM extending the Hooper et al. analytical model onto Village retains all the agent actions in Village but adds a public goods game. When the simulation begins, in AD 600, 200 agents are randomly placed on the landscape, as in the ordinary version. Each is composed of two parents and a variable number of children appropriate to the ages of the parents and the life table we employ. Parents in the households are randomly divided between H and NH types; H types are randomly assigned to one of the 8 H types in Table 2, and NH types are randomly assigned to one of the 3 NH types in Table 2. Children inherit their type from their mother. Potential leaders are randomly assigned a tax rate between the minimum and maximum values identified in Table 2. Households act according to the type of the mother if she is alive; if she is not they act according to type of the father. As families reproduce, descendant families will tend to share their strategy traits with their ancestors’ other descendents by virtue of common descent. This endogenous relatedness could affect the trajectory of trait evolution in the population, though we have not tried to determine whether there is sufficient positive assortment of relatives to affect the model’s results, relative to a truly kinship-less scenario.

Households can exist in three states: 2 (thriving), 1 (just getting by), and 0 (perishing). The default value for state is 2, but this gets lowered to 1 if the current maize in storage is less than that needed for the current year plus that expected to be needed for next year; or if the maize just harvested is less than next year’s anticipated needs (taking into account the ages and sexes of the family members). Households in state 1 reproduce according to a life table that provides for an approximately stable global population. The probabilities of giving birth are incremented by 10% for women in a household in state 2, and the probabilities of dying are decremented by 10% for members of that household. The converse happens for households in state 0 (for details, see [14b]). A household’s strategy for playing the public goods game affects its maize storage and perhaps its state, and may therefore increase, or decrease, its relative number of offspring, who inherit the strategy, providing a slow evolutionary dynamic to strategy change in the population.

3.3. Group formation

Groups are formed within a distance determined by the parameter MAX_COOP_RADIUS_BRN, which dictates the maximum distance at which balanced reciprocal exchanges may take place, one-half the maximum distance that may separate agents within a group, and the maximum distance at which agents can influence others through social learning (discussed below). As shown in Table 3, we examine the effects of varying this by increments of 10 from 30 to 50 (cells, where 30 cells = 6 km). Because of the toroidal world, the highest value results in groups spread more or less across the entire model landscape.

For group formation, agents are filtered into two sets, H and NH. H groups are formed first. Potential leaders are processed in order of tax rate (lowest rate
T. A. Kohler et al.

Table 3. Parameters varied in the 36 runs reported here.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX_CPBRN</td>
<td>Maximum radius for interhousehold balanced reciprocal exchanges,</td>
<td>30, 40, 50 cells (6, 8, and 10 km)</td>
</tr>
<tr>
<td>CMﻚ MONITOR C205</td>
<td>Cost of monitoring one group member</td>
<td>2, 4, 6 days kg</td>
</tr>
<tr>
<td>GROUPenefitGrowth_Rate</td>
<td>Exponent controlling rate of growth of positive returns to cooperation</td>
<td>2, 5</td>
</tr>
<tr>
<td>CHANGE_RATE</td>
<td>Strength of social learning</td>
<td>0.0, 0.2</td>
</tr>
</tbody>
</table>

first), and try to form groups out of the ungrouped H agents within the radius set by MAX_CPBRN. This ensures that any two members of the created group will be within range of each other. Once the group is formed, it will elect as leader that agent offering the lowest tax rate, which would in this case be the founder of the group. While a group can only have one leader, there may be many other potential leaders within the group, even though they would act the same as any other non-leading member. Groups check every year to see whether a new less costly leader might be available. If an active leader dies, the group tries to replace it immediately, disbanding if they cannot.

NH groups are formed by NH agents; a randomly chosen agent scans the permissible range for other NH agents not yet in groups, as well as for H agents that were not within range of a leader when the H groups were formed. When MAX_CPBRN is 40 this process results in about 13 H and 16 NH groups (which may include leaderless H agents) at the beginning of the simulation. (Smaller radii lead to more groups, and vice versa.) H and NH groups may overlap in space, and often do. In some cases, H groups and NH groups can also overlap with other groups of the same type. This is due to the requirement that agents are within range of every other member of the group. Visual inspection of running simulations suggests that overlap among groups increases with larger radii for MAX_CPBRN, but we have not attempted to quantify this effect.

The underlying machinery of Village dictates that agents will try to move if their current location ceases to provide them with amounts of the four resources it needs, obtainable within the reasonable time and caloric constraints that we impose. Agents do not consider whether a move will take them outside the range of their present group. When agents move they are accommodated into the closest group of matching type (H, NH), if one is available. Moves are most common at the beginning of the simulation, and in periods of high climatic variability. When individuals marry they form new households on the best location within a parametrized radius of either parent’s household, here set to 10 cells (2 km). This may take them outside the range of the parents’ group, and in fact the parents may be from different groups.
This new household will behave as any other moved household, and try to join an existing group (or form one of its own if it needs to).

3.4. Playing the public goods game

The public goods game happens once per year, at the end of the winter activities. All costs and benefits from the game are expressed in calories, or equivalently in maize, which is the source of calories in the model. A punished defector, for example, has its maize storage decremented by an appropriate amount.

The leader of each H group collects contributions from each member household. The amount of the contribution is determined by the parameter $c_{cost}$. If a household refuses or is unable to pay the full contribution, it is punished by the leader, who incurs a cost to do this. As agents will still be punished even if they could pay 90% of the contribution, they will pay none of it if they cannot pay the full amount. H agents that refuse to contribute to the public good or pay taxes and are subsequently punished will not defect again in the group in which they were punished, but will revert to defection if they move to a new group. In addition to the costs of punishing defectors, leaders also incur costs to monitor the behavior of group members. Group leaders also contribute to the public good, and never defect, in spite of the fact that some may have done so if they were not the leader (for example if they are of type H.RC.RT.L).

Before the leaders distribute the amount collected, it is incremented by a factor that represents positive returns to group size, where group size is calculated as the contributors to the public good. In the real world, the source of this increment may come from any of a number of the group-beneficial activities introduced earlier (domestic water from a reservoir; the tangible or reputational benefits accruing from participating in ceremonial activities; etc.). For the purposes of the present simulation, however, it is left as a black box. Finally, the leader distributes this enhanced pool of maize, dividing the benefit equally among all members of the group, regardless of whether they paid their contribution. This means that the size of the benefit returned is a function of both the number of contributing households and the total number of households. The size of the benefit generated by each contributing household is calculated as:

$$KG_{DAY} \times b_{benefit} \times e^{-\left(\frac{Group_{benefit}_{growth \ rate}}{n_{contributors}}\right)},$$

(4)

where $KG_{DAY}$ is a constant (1.53) converting days to kg of maize, $b_{benefit}$ is a parameter of the simulation, here set to 73 days kg, representing the base benefit produced by contributing to the public good, $n_{contributors}$ is the number of people in the group contributing to the public good, and $Group_{benefit}_{growth \ rate}$ is a parameter to the simulation, for which we explore values of 2 and 5, that affects how quickly the benefits of group size reach diminishing returns. The lower the $Group_{benefit}_{growth \ rate}$, the more quickly diminishing returns are reached.
T. A. Kohler et al.

For example, if both group size and \( n_{\text{contributors}} = 30 \), \( \text{Group}\_\text{benefit}\_\text{growth}\_\text{rate} = 2 \), and the tax rate \( t = 0.2 \), (4) yields 105 kg per household, from which 21 kg would be subtracted as tax, leaving a benefit of 84 kg maize. Assuming that all households contributed their cost of 37 days kg (which converted to kg maize by multiplying by KG\_DAY [1.53] = 56.6 kg maize), each household would net 48% return (84/56.6) on their investment in the public good. Net returns to contributors of course decline with increasing numbers of defectors, higher tax rates, and higher \text{Group}\_\text{benefit}\_\text{growth}\_\text{rates}. Table 4 gives values for \( e^{(-\text{Group}\_\text{benefit}\_\text{growth}\_\text{rate}/n_{\text{contributors}})} \) for the two values of \text{Group}\_\text{benefit}\_\text{growth}\_\text{rate} we employed and for relatively small numbers of \( n_{\text{contributors}} \). The functional form of expression (4) causes the slope of this function to decrease as \( n \) increases, capturing an intuition that in many peaceful pursuits the net marginal value of increasing group size tends to decrease as overhead for maintaining coordination increases. If our chief interest were returns on increasing group size in inter-group conflict, we might choose a different functional form.

Figure 4 (top) compares the gross individual benefits and the total group benefits expected from the public goods game as a function of total group size and fraction of group members that contribute, as played here. We contrast that with payoffs from the classic public goods game (below). With a total group size of 25 and \text{Group}\_\text{benefit}\_\text{growth}\_\text{rate} of 5, for example, 70% of the group would have to contribute to get at least 37 days kg in benefits (the break-even point for households). Even if everyone in the group contributes, the total group size must be at least 8 households to get more than 38 days kg in benefits.

As this figure illustrates, the benefits function yields slightly increasing returns to scale, but converges on constant returns to scale for large groups. The public goods game will only yield net benefits if groups are relatively big and have a high fraction of contributors.

Leaders also collect taxes, which are a percentage of the benefit received by each agent from the public goods game. The leader asks each member agent to pay its tax (which is the same for each household) immediately after the benefit from the public goods game is distributed. If an agent is an unpunished RT, it will refuse, in which case it is punished by the leader, who again incurs a cost to do this.

In NH groups all monitoring and punishment actions are performed by Mutual Monitor (MM) agents. Each MM incurs a cost to monitor the behavior of all other

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<th>\text{Group}_\text{benefit}_\text{growth}_\text{rate}</th>
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Fig. 4. Gross household (left) and total group (right) benefits expected from the public-goods game as a function of group size and fraction of contributing group members. (a) as played here, with `GROUP\_BENEFIT\_GROWTH\_RATE` = 5; (b) as played here, with `GROUP\_BENEFIT\_GROWTH\_RATE` = 2; (c) as in the classic public-goods game.
group members, including other MMs. If a group member refuses to pay its contribution, it is punished by all the MMs in the group, each of whom pays the full punishment cost. The punished agent thus gets charged the sanction \( s \) times the number of MMs. The amount of benefits the group will receive is calculated in the same way as with H groups, based on the sum of the contributions, modified by the positive returns to scale factor described above. The benefit is divided equally among all members of the group, regardless of any defections. In all cases for NH groups, defectors will not repeat defections once they have been punished.

3.5. **Social learning**

In Hooper *et al.* [12], leadership emerges over dozens or hundreds of generations; exactly how many depends on the parameter values, the starting frequencies, and especially on the size of \( V_0 \), the baseline fitness, which affects the strength of selection. More successful strategies reproduce in greater numbers, leading eventually to more of their offspring being present in the population. As a Village is based on a human society, reproduction is a slow process. For the entire run of our simulation, there are only some 35 generations (700 years/20 years per generation). In keeping with real human societies, we introduce a simple social learning model to augment the reproductive model for strategy evolution.

We base this on the social learning theories developed originally by psychologists such as Julian Rotter and Albert Bandura. Social learning theory has since been elaborated, formalized, and promulgated among anthropologists as cultural transmission theory, largely by Boyd and Richerson [8] and their students.
In our simulation, each agent possesses a type scale between 0 and 1 that dictates its willingness to work in societies with a leader. NH agents are initialized with a random value between 0 and 0.5, while H agents begin with a value randomly varying between 0.5 and 1. After each iteration of the public-goods game, each agent locates the richest agent (the household with the most stored maize) within \( \text{max}_\text{coop}_\text{radius}_\text{brn} \). The focal agent will then shift its type scale toward the type of this more successful agent. The amount of the shift is a parameter of the simulation \( (\text{change}_{\text{Rate}}) \) which is here set to either 0 (no social learning) or 0.2 (Table 3). If the scale crosses the 0.5 threshold, the agent changes its behavioral type. If it happens that the model agent is of the same type, then that type is reinforced. When the \( \text{change}_{\text{Rate}} = 0.2 \), an agent will take at most 3 years to change its preference for living in a hierarchical group if its model is consistently of the other type.

Because H and NH agents have different attributes, when agents change types, their other behaviors relevant to the public-goods game may change as well. In conversion from H to NH, ALLC and RC retain that behavior, and H.RC.T converts to NH.MM. Conversions in the opposite direction are a little more complicated because H types use more bits in their description than do NH types. In conversions from NH to H, MMs convert to RC so as not to give an undue advantage to hierarchical groups. Their other attributes (T/RT, L/UL, and tax rate) are randomized (within the bounds specified in Table 1 for tax rate). This decision, once again, was made so as to not privilege some specific outcome and to add some degree of mutation-like variability to the system. Quite logically, NH.ALLC converts to H.ALLC, and NH.RC converts to H.RC, in each case with the other bits randomized.

Two features of our representation of social learning deserve comment. First, archaeologists might object that it is not realistic for households to observe or accurately estimate the amount of maize stored by other households, since in our reference setting storage is typically held in a concealed area such as a back room. We argue that so many signals of household wealth would be visible in these small-scale societies (for example the number and size of its fields, the success of their crops, its willingness to respond to requests for exchange, and its number of children) that a household’s amount of storage would be private in name only.

Second, the fact that our agents emulate only the propensity of a model household to work in a hierarchical setting, and not its other behaviors related to the public-goods game, means that the behavior most responsible for an agent’s success is not necessarily the behavior that is imitated. Ours then is a model of indirect bias as defined by Boyd and Richardson [8, pp. 241–259].

4. Results

Figure 5 displays the average number of hierarchical and nonhierarchical agents produced in 36 runs of Village with the parameters shown in Tables 1 and 3, dividing these agent types by runs in which social learning was, and was not, enabled.
The slow “genetic” dynamic is sufficient, for our parameter choices, to produce, on average, at least three to four as many H agents as NH agents in the later years of these 700-year simulation runs; adding social learning increases this ratio only slightly.

Figure 6 shows the numbers of H and NH groups across these radii classes, and Fig. 7 shows the histories of the average size of H and NH groups as we vary $\text{MAX}_\text{COOP}_\text{RADIUS}_\text{BRN}$. A critical prediction of the Hooper et al. model is that H strategies should be able to successfully carry out collective action in larger groups than NH strategies, and this effect is obvious in our implementation of this model, even after the first 100 years. Matched for values of $\text{MAX}_\text{COOP}_\text{RADIUS}_\text{BRN}$, and averaging across the other parameters, H groups tend to be five to six times larger than NH groups through the last 300 years of the simulation. The relatively greater instability in sizes of H groups than NH groups is visually obvious from Fig. 7, in which the points marking the average sizes in each year are far more separated for the H than for the NH groups. The large H groups that form when $\text{MAX}_\text{COOP}_\text{RADIUS}_\text{BRN} = 50$ are especially unstable, particularly after the mid-AD 1000s. In these runs it will often happen that there is an oscillation between 4 or 5 groups, or between 5 and 6 groups; in the context of large group sizes this causes great variability in mean agents/group.
Fig. 6. Mean number of hierarchical (H) and non-hierarchical (NH) groups across three levels of $\text{max}_\text{coop} \text{ radius}_{\text{brn}}$.

Fig. 7. Mean households per hierarchical (H) group and non-hierarchical (NH) group across three levels of $\text{max}_\text{coop} \text{ radius}_{\text{brn}}$. 
The spatial dynamics of the simulation are unfortunately invisible in these graphs and difficult to impart in a journal. Spatial behavior is quite sensitive to the settings for parameters in the public goods game as well as in the environment in which that operates, and we will report on these complications elsewhere. At the beginning of the simulation, agents with H and NH preferences are equally likely to live in areas of high potential maize productivity. In many parameter combinations, NH groups on good patches however soon grow to a size where mutual monitoring becomes costly. As monitors globally decrease in frequency, if some groups have none then defecting members of NH groups remain unpunished and returns on the public goods games in these groups decline. NH group members near H groups may be attracted to join these groups, given their generally higher prosperity. In poorer areas, especially in the western and south-central parts of our area, NH groups more rarely grow to a size where their organization becomes disadvantageous.

The number of H groups is little affected by max\_coop\_radius\_brn, and for all three of the values tried, H group counts regularly decrease through time as their sizes generally increase. By the end of the simulation, there are only 6–7 H groups remaining on the landscape regardless of the value for max\_coop\_radius\_brn. On the other hand, NH group counts are quite sensitive to this parameter, with many more NH groups present as values of max\_coop\_radius\_brn decrease.

Figure 8 shows the mean counts for agent types across all 36 runs. In general, NH types decline in frequency through time relative to all agents. NH.MMs, which are...
the most successful type for the first 200 years on average, decline sharply in number after NH groups reach an average size of some 30–40 households (Fig. 6). The most successful types in the long run are H.RC.T.UL and H.RC.RT.UL, followed closely by the leader type H.ALLC.RT.L.

The tax rates charged by active (elected) leaders in all 36 runs are in Fig. 9. The points marking rates for all active leaders in each year exhibit a high degree of variability. The means for all the three radii slowly increase through about AD 800, after which growth slows or stops. The lower tax rates in the radius-50 runs is presumably the result of more competition among potential leaders. In this light it is surprising that the radius-40 leaders are able to average slightly higher tax rates than the radius-30 leaders. This may be due to some subtle interaction between these radii and the structure of production on this landscape.

For one run (where change\_rate = 0.2, cm\_monitor\_cost = 6, max\_coop\_radius_brn = 30, and group\_benefit\_growth\_rate = 5) we graph the returns on the public-goods game for each agent type (Fig. 10). In general, leaders do fairly well, though their benefits can vary greatly through time; NH types on average get negative returns from the game. One type (H.ALLC.T.UL) becomes extinct in this run, in the mid-12th century. Figure 8 shows that this is the least-successful H type overall, though in most other parameter combinations it outperforms the NH types by the end of the simulation.
Fig. 10. Mean returns (in kg maize) from the public-goods game by agent type through time in one run.

5. Discussion and Conclusion

To the extent that the costs and benefits we have chosen for participation in the public-goods game are realistic for our setting, we have demonstrated that the advantages of living in groups with a hierarchical structure are sufficient to allow such groups to grow in size, and their members to grow in number, marginalizing non-hierarchical groups on the landscape. This growth takes place through demographic processes even in the absence of social learning. With a simple scheme of social learning the disparities between the two types of organization are slightly magnified as hierarchical groups attract recruits from non-hierarchical groups.

Formal (mathematical) models of social processes, such as that provided by [12], have the advantage of being extremely general, and therefore potentially applicable to a variety of settings. The flip side to such generality is that it is difficult to specify how accurately such models reference any particular empirical sequence. We suggest that a rigorous examination of such a model must entail exactly the process we demonstrate here — instantiating it in a particular setting to determine the extent to which it can explain the changes inferred in that setting by other means.

As briefly reviewed above, the first hints of leadership in our area appear in the mid-late AD 700s, and become more strongly expressed by the mid-800s [13, 18, 33].
This is nicely in keeping with the results of the model, which typically show the first significant divergence in the sizes of hierarchical and nonhierarchical groups in the mid-700s (Fig. 7). Tax rates for leaders (Fig. 9) have increased enough by the mid-700s to make leadership attractive, especially in conjunction with the sharply increasing sizes for hierarchical groups from 700 to 900.

The simulations and the real VEP trajectory diverge significantly after about AD 900, when VEP-area populations collapsed along with any evidence of hierarchical organization. This does not happen in the model, possibly because the production estimates on which it is based do not take into account the effects of low-frequency trends towards much colder and somewhat drier conditions from the late 800s through about 1000 [14a, Fig. 5.4]. When a wave of immigrants around AD 1080 markedly increased VEP-area populations, evidence for hierarchy (derived from or inspired by the Chacoan regional system, which by now was re-centered at Aztec near the present Farmington, NM) re-appeared as well. The large Chacoan centers were in the eastern and central portions of the VEP area, and generally absent from the western portions, in keeping with the main concentrations of hierarchical and non-hierarchical populations in the model by this time.

The model and reality again diverge significantly in the late-AD 1200s. VEP populations were in sharp decline after about AD 1250, and by about 1280 farming populations had entirely vacated the VEP area, and in fact all of the northern Southwest. Agent populations on the other hand decline only slightly during this period (Fig. 8). Once again, we consider it likely that this divergence is due primarily to the insensitivity of our paleoproduction estimates — based on the high-frequency signals in the tree-ring data — to the low-frequency cooling and drying trends recently reconstructed for this area from pollen data [39; 14a, Fig. 5.4].

As we noted at the beginning of this paper, our goal was to model one pathway for the emergence of leadership in small-scale societies. Nothing we have said eliminates the likelihood that other pathways exist. For example, local leaders could arise to interact with representatives of more complex groups farther away. Such a model though would need to specify why having a leader would make relationships with other groups more successful; what costs leaders and other group members incur as a result of the new social arrangement; and how the net benefits from having a leader are distributed among the leader and other group members. As such, many of the dynamics captured in our model would be relevant to understanding the emergence of hierarchy to facilitate inter-group relations, even if the initial incentives motivating the transition to leadership are different from those in our public-goods game.

Although we are pleased by the successes reported here, and can understand the reasons for some of the periods of lack-of-fit that we note, we recognize three main issues to be addressed to make this a more satisfactory model for sociopolitical processes in our area.

First, this is a voluntaristic model for the development of leadership, rather than a coercive one; we have modeled “elites-as-managers” rather than
“elites-as-thugs” [2]. Following [35], we might think of these leaders as more “bashful” than “boastful”. A voluntaristic model such as this neglects, by design, hostile intergroup competition as a driver for forming and maintaining hierarchies. Though some group fissioning may emerge in our model through inter-leader competition, warfare should have the effect of favoring larger groups, since, other factors equal, larger groups prevail in conflict. A time series for social strife in and around our area, produced from data on human skeletal remains [9, 16], suggests that violence was a relatively minor part of the social fabric until the late-AD 800s, with a small peak in the early 900s, followed by near-cessation of violence. However, the Chacoan entry into our area in the mid-late 1000s, and (especially) the break-up of the Chacoan system in the mid-1100s, were very violent. The early 1200s marked a return to low levels of strife, which again increased rapidly as the depopulation began [22].

Comparison of this record with our voluntaristic model suggests that the early appearance of leadership in our area can be explained by voluntaristic processes. The larger group sizes, and greater evidence for hierarchy and violence in the Chacoan and immediately post-Chacoan periods, however, may require a model that explicitly incorporates inter-group competition and a hierarchy of leaders, following models of Turchin and Gavrilets [36] and in accord with empirical demonstration of hierarchies in Chaco great house sizes in the eastern Southwest by [29, p. 313].

Finally, a third avenue for model development is to more fully endogenize the public-goods game. Instead of modeling it as a black box, we could model specific cooperative activities in which there are positive returns to scale, such as long-distance large-game hunting, building reservoirs, or inter-group warfare. Even as it stands we hope that our model will interest archaeologists in the task of assessing the probable costs and benefits of such interactions, since the success of this model implicates these interactions in the origins of political inequality in the US Southwest and, by plausible extension, Neolithic societies worldwide.

Acknowledgments
We thank Anne Kandler and James Steele for their kind invitation to speak in the UCL conference “Cultural Evolution in Spatially Structured Populations” in September 2010. This paper would not have been possible without the contributions

\[\text{Voluntaristic hierarchy formation (and with it bashful “elites-as-managers”) could also emerge in the context of intergroup competition, and “elites-as-thugs” could also occur without inter-group competition (for example, as wealth inequalities build up and the richest individuals begin to force other group members into debt slavery). So we imagine a } 2 \times 2 \text{ cross-tabulation of the possibilities as follows: (1) elites-as-managers in the context of peaceful collective action; (2) elites-as-managers in the context of collective action in the form of warfare; (3) elites-as-thugs who assert dominance within groups (effective primarily if there are constraints on emigration); and (4) elites-as-thugs who assert dominance over other groups. Here we have explored box 1. The public-goods game here could be modified without too much trouble to explore box 2 as well; exploring boxes 3 and 4 would require some different machinery.}\]
of many other past and current members of the VEP research team, especially Mark Varien and Scott Ortman of Crow Canyon Archaeological Center. Thanks to Herb Maschner, Scott Ortman, and an anonymous reviewer for comments on an earlier version. Hillard Kaplan and Jim Boone also provided helpful input. Stefani Crabtree produced Fig. 1, and Ruth Van Dyke provided a crucial reference. Our research is based on work supported by the National Science Foundation under Grant DEB-0816400.

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T. A. Kohler et al.


